# SIMULATION AND ANALYSIS OF TRANSPORT OF GASEOUS WASTES FROM AUTOMOBILE TRAFFIC WITH RANDOM CHARACTERISTICS 

M. G. Boyarshinov

UDC 519.2:536.24


#### Abstract

An approximate model of transport and dispersion of automobile waste gases by a transverse air flow is considered. The motion of cars is assumed to be a random Poisson process. One possible approach to estimating the concentration of the wind-transported waste and determining its statistical characteristics is described. The length of the representative section of a roadway that is sufficient for a correct analysis of the ecological situation is determined. The power of a linear stationary source approximating the distribution of the mathematical expectation of the concentration of wastes exhausted by a random flux of cars is evaluated.


The solution of the problem of estimating contamination of the atmosphere near automobile routes is complicated by the fact that the appearance of the next car on the road is conditioned by numerous random factors. Additional difficulties arise due to the unsteadiness of the process of transport and dispersion of waste products exhausted by a random flux of cars by the moving atmospheric air.

The contribution of the maximum wastes of automobile transport to the overall level of contamination of the atmosphere of a modern city was estimated by Rodivilova et al. [1]. The effect of automobile-induced contamination on plants and vegetables adjacent to roadways with intense traffic was considered by Tarankov and Matveev [2]. Volkova and Samoilova [3] developed a model that allows one to study the dependence of the concentration of pollutants, the degree of contamination, and the size of the polluted zone on the traffic intensity, composition of the transport flux, road parameters, and meteorological factors.

The concentration of waste products exhausted by automobile traffic is most often estimated using models of straight-line stationary sources. The model of [4] is applicable if the length of the road sector considered is many times greater than the distance from the control point; if this condition is not satisfied, the concentration is overrated, as is noted in [5]. The model of [6] is used for finite-length sources and is applicable if the wind direction is perpendicular to the road. The wind direction is taken into account in the model [7] of a straight-line, finite-length source of waste products.

The concentrations of $\mathrm{CO}, \mathrm{NO}_{\mathrm{x}}$, and solid particles in exhaust gases of different types of cars were estimated Luhar and Patil [8]. The transport of wastes near car routes was simulated in a wind tunnel by Heidorn et al. [9]. The measured results were compared with the calculated data; it was noted that the reason for the difference observed is the inaccurate description of the shape of buildings in the theoretical model. Using two- and three-dimensional mathematical models, Moriguchi and Uehara [10] calculated the air-flow velocities and automobile-induced waste concentrations along city roadways. Kasibhatla et al. [11] used the finite-element method to simulate the transfer of waste products in the atmosphere from a road approximated by an infinite source.

In the present paper, we consider a possible approach to estimating the field of waste concentration near a roadway using an approximate solution of the problem on transportation of automobile wastes by

Perm' State Technical University, 614600 Perm'. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 41, No. 6, pp. 86-94, November-December, 2000. Original article submitted February 23, 2000; revision submitted April 24, 2000.


Fig. 1. Layout of the problem of motion of an automobile flux.
atmospheric flows. Let a spatial domain $G$ contain an extended (in the direction of the $O y$ axis) sector of a one-way, single-lane road of length $L$ (Fig. 1). We assume that the velocity of all cars moving on this road is identical, constant, and equal to $V$. The road experiences the action of a horizontal air flow with a constant velocity $U$ directed at an angle $\alpha$ to the $O x$ axis. The air-flow velocity at all points of the domain $G$ is assumed to be independent of the positions, velocities, and characteristics of cars. The concentrations of waste products near the road depend on the volume of wastes exhausted by all cars that are simultaneously located on the road sector considered and are moving point sources of contamination with a constant intensity $q$. The appearance of a car at the beginning of the road sector considered is a random Poisson flux of events with a constant intensity $\lambda$.

We have to find the distribution of the concentration of wastes exhausted by the cars in the domain $G$ and the finite length $L$ of the road sector (characteristic size) sufficient for determining the concentration of wastes at the control point. We also have to evaluate the validity of replacing a three-dimensional problem by a two-dimensional scheme of propagation of wastes from a straight-line stationary source.

We consider an unsteady propagation of wastes from moving sources. The use of the moving coordinate system $O^{\prime} x^{\prime} y^{\prime} z^{\prime}$ fitted to a car moving with a constant velocity allows us to pass to the problem described by a steady equation [12] of diffusion of pollutants from a point source located at a point with coordinates $x^{\prime}=0, y^{\prime}=0$, and $z^{\prime}=0$ (Fig. 1):

$$
\begin{equation*}
U_{x}^{\prime} \frac{\partial \varphi}{\partial x}+U_{y}^{\prime} \frac{\partial \varphi}{\partial y}+U_{z}^{\prime} \frac{\partial \varphi}{\partial z}=\frac{\partial}{\partial x}\left(K_{x} \frac{\partial \varphi}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{y} \frac{\partial \varphi}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{z} \frac{\partial \varphi}{\partial z}\right)+q \delta\left(x^{\prime}, y^{\prime}, z^{\prime}\right) \tag{1}
\end{equation*}
$$

Here $\varphi$ is the concentration of pollutants, $\delta$ is the Dirac delta function, and $K_{x}, K_{y}$, and $K_{z}$ are the coefficients of turbulent diffusion. The relative velocity of the air flow in the moving coordinate system has the following components:

$$
\begin{equation*}
U_{x}^{\prime}=U \cos \alpha, \quad U_{y}^{\prime}=U \sin \alpha-V, \quad U_{z}^{\prime}=W \tag{2}
\end{equation*}
$$

Assuming that the wind velocity is independent of the height and the coefficients $K_{x}, K_{y}$, and $K_{z}$ are constant, the solution of Eq. (1) has the form [12]

$$
\begin{gather*}
\varphi\left(x^{\prime}, y^{\prime}, z^{\prime}\right)=q \exp \left(\frac{U_{x}^{\prime} x^{\prime}}{2 K_{x}}+\frac{U_{y}^{\prime} y^{\prime}}{2 K_{y}}+\frac{U_{z}^{\prime} z^{\prime}}{2 K_{z}}-\frac{1}{2} \sqrt{\left.\frac{x^{\prime 2}}{K_{x}}+\frac{y^{\prime 2}}{K_{y}}+\frac{z^{\prime 2}}{K_{z}} \sqrt{\frac{U_{x}^{\prime 2}}{K_{x}}+\frac{U_{y}^{\prime 2}}{K_{y}}+\frac{U_{z}^{\prime 2}}{K_{z}}}\right)}\right. \\
/\left(4 \pi \sqrt{K_{x} K_{y} K_{z}} \sqrt{\frac{x^{\prime 2}}{K_{x}}+\frac{y^{\prime 2}}{K_{y}}+\frac{z^{\prime 2}}{K_{z}}}\right) . \tag{3}
\end{gather*}
$$

In the stationary coordinate system $O x y z$, taking into account (2) and the dependences $x^{\prime}=x$, $y^{\prime}=y-y_{i}$, and $z^{\prime}=z-b$, we transform solution (3) to

$$
\begin{gather*}
\varphi^{+}\left(y_{i}, x, y, z\right)=q \exp \left(\frac{x U \cos \alpha}{2 K_{x}}+\frac{(U \sin \alpha-V)\left(y-y_{i}\right)}{2 K_{y}}+\frac{W(z-b)}{2 K_{z}}\right. \\
\left.-\frac{1}{2} \sqrt{\frac{x^{2}}{K_{x}}+\frac{\left(y-y_{i}\right)^{2}}{K_{y}}+\frac{(z-b)^{2}}{K_{z}}} \sqrt{\frac{U^{2} \cos ^{2} \alpha}{K_{x}}+\frac{(U \sin \alpha-V)^{2}}{K_{y}}+\frac{W^{2}}{K_{z}}}\right) \\
/\left(4 \pi \sqrt{K_{x} K_{y} K_{z}} \sqrt{\left.\frac{x^{2}}{K_{x}}+\frac{\left(y-y_{i}\right)^{2}}{K_{y}}+\frac{(z-b)^{2}}{K_{z}}\right) .}\right. \tag{4}
\end{gather*}
$$

Here $y_{i}$ is the position of the moving source is the coordinate system Oxyz. Taking into account that $y_{i}=V t_{i}-0.5 L$ ( $t_{i}$ is the time of motion of the source from the moment of its appearance on the road), we can determine this concentration as a function of time:

$$
\begin{gather*}
\varphi^{+}\left(t_{i}, x, y, z\right)=q \exp \left(\frac{x U \cos \alpha}{2 K_{x}}+\frac{(U \sin \alpha-V)\left(y+0 / 5 L-V t_{i}\right)}{2 K_{y}}+\frac{W(z-b)}{2 K_{z}}\right. \\
\left.-\frac{1}{2} \sqrt{\frac{x^{2}}{K_{x}}+\frac{\left(y+0.5 L-V t_{i}\right)^{2}}{K_{y}}+\frac{(z-b)^{2}}{K_{z}}} \sqrt{\frac{U^{2} \cos ^{2} \alpha}{K_{x}}+\frac{(U \sin \alpha-V)^{2}}{K_{y}}+\frac{W^{2}}{K_{z}}}\right) \\
/\left(4 \pi \sqrt{K_{x} K_{y} K_{z}} \sqrt{\frac{x^{2}}{K_{x}}+\frac{\left(y+0.5 L-V t_{i}\right)^{2}}{K_{y}}+\frac{(z-b)^{2}}{K_{z}}}\right) . \tag{5}
\end{gather*}
$$

To take into account the condition of the absence of waste flow through the solid surface $\partial \varphi\left(y_{i}, x, y, 0\right) / \partial z=0$, we introduce a fictitious source located symmetrically to the initial one with respect to the horizontal plane $z=0$; the concentration of pollutants from this source is determined by the expression

$$
\begin{gathered}
\varphi^{-}\left(y_{i}, x, y, z\right)=q \exp \left(\frac{x U \cos \alpha}{2 K_{x}}+\frac{(U \sin \alpha-V)\left(y-y_{i}\right)}{2 K_{y}}-\frac{W(z+b)}{2 K_{z}}\right. \\
\left.-\frac{1}{2} \sqrt{\frac{x^{2}}{K_{x}}+\frac{\left(y-y_{i}\right)^{2}}{K_{y}}+\frac{(z+b)^{2}}{K_{z}}} \sqrt{\frac{U^{2} \cos ^{2} \alpha}{K_{x}}+\frac{(U \sin \alpha-V)^{2}}{K_{y}}+\frac{W^{2}}{K_{z}}}\right) \\
/\left(4 \pi \sqrt{K_{x} K_{y} K_{z}} \sqrt{\left.\frac{x^{2}}{K_{x}}+\frac{\left(y-y_{i}\right)^{2}}{K_{y}}+\frac{(z+b)^{2}}{K_{z}}\right) .}\right.
\end{gathered}
$$

The solution is constructed in the form $\varphi\left(y_{i}, x, y, z\right)=\varphi^{+}\left(y_{i}, x, y, z\right)+\varphi^{-}\left(y_{i}, x, y, z\right)$ or $\varphi\left(t_{i}, x, y, z\right)=$ $\varphi^{+}\left(t_{i}, x, y, z\right)+\varphi^{-}\left(t_{i}, x, y, z\right)$. The total concentration $\Phi$ of pollutants at an arbitrary point $(x, y, z)$ from a random number $\tilde{N}$ of cars located on the examined road sector of length $L$ can be determined as a function of the coordinates $y_{i}$ of cars or the time $t$ :

$$
\begin{equation*}
\Phi(x, y, z)=\sum_{i=1}^{\tilde{N}} \varphi\left(y_{i}, x, y, z\right), \quad \Phi(x, y, z)=\sum_{i=1}^{\tilde{N}} \varphi\left(t-T_{i}, x, y, z\right) . \tag{6}
\end{equation*}
$$

Here $T_{i}$ is the moment of appearance of the $i$ th car at the beginning of the road.
Dependences (6) allow us to determine the mathematical expectation $M_{\Phi}=\langle\Phi\rangle$ and the root-meansquare deviation $\sigma_{\Phi}=\sqrt{\left\langle[\Phi-\langle\Phi\rangle]^{2}\right\rangle}$ for an arbitrary point of the domain $G$. The intervals $\Delta t$ between the moments of appearance of cars $T_{i}$ are distributed in accordance with the law $p(\Delta t)=\lambda \exp (-\lambda \Delta t)$. The mathematical expectation of the distance $\Delta y=V \Delta t$ between the cars on the road is

$$
\langle\Delta y\rangle=\int_{0}^{\infty} \Delta y p(\Delta y) d(\Delta y)=\frac{\lambda}{V} \int_{0}^{\infty} \Delta y \exp \left(-\frac{\lambda \Delta y}{V}\right) d(\Delta y)=\frac{V}{\lambda} .
$$

The length of the car column is determined by the sum of a random number of random quantities $\tilde{L}=\sum_{i=1}^{\tilde{N}} \Delta y_{i}$. Hence, the mathematical expectation of its length is $\langle\tilde{L}\rangle=\left\langle\sum_{i=1}^{\tilde{N}} \Delta y_{i}\right\rangle=\langle\tilde{N}\rangle\langle\Delta y\rangle$. The statistical mean number of cars on the road is $N=\langle\tilde{N}\rangle=\langle\tilde{L}\rangle /\langle\Delta y\rangle=\lambda L / V$. Since we consider a stationary Poisson [13] flux of cars, the relation $D[\tilde{N}]=\langle\tilde{N}\rangle=L \lambda / V$ is valid. Taking into account Eq. (6), we find the mathematical expectation of the concentration and its root-mean-square deviation at the control point:

$$
\begin{gather*}
M_{\Phi}(x, y, z)=\left\langle\sum_{i=1}^{\tilde{N}} \varphi\left(y_{i}, x, y, z\right)\right\rangle=\langle\tilde{N}\rangle\left\langle\varphi\left(y_{i}, x, y, z\right)\right\rangle=\frac{\lambda}{V} \int_{-0.5 L}^{0.5 L} \varphi\left(y_{i}, x, y, z\right) d y_{i} ;  \tag{7}\\
\sigma_{\Phi}(x, y, z)=\sqrt{\langle\tilde{N}\rangle D\left[\varphi\left(y_{i}, x, y, z\right)\right]+D[\tilde{N}]\left\langle\varphi\left(y_{i}, x, y, z\right)\right\rangle^{2}}=\sqrt{\frac{\lambda}{V} \int_{-0.5 L}^{0.5 L} \varphi^{2}\left(y_{i}, x, y, z\right) d y_{i} .} \tag{8}
\end{gather*}
$$

For a known distribution density of the probability of the wind direction $p(\alpha)$, the mathematical expectation of the waste concentration is calculated by the formula

$$
\begin{equation*}
M_{\Phi}^{\alpha}(x, y, z)=\frac{\lambda}{V} \int_{-0.5 L}^{0.5 L} \int_{0}^{2 \pi} \varphi\left(y_{i}, x, y, z\right) p(\alpha) d \alpha d y_{i} \tag{9}
\end{equation*}
$$

Expressions (7)-(9) are independent of time, which indicates the steadiness of the random process of gaseous wastes from motor transport entering an arbitrary point of the domain $G$ under consideration. For a track with multilane traffic (or cars of different kind), the expressions for the mathematical expectation and root-mean-square deviation have the form

$$
M_{\Phi}(x, y, z)=\sum_{i} \frac{\lambda_{j}}{V_{j}} \int_{-0.5 L}^{0.5 L} \varphi_{j}\left(y_{i}, x, y, z\right) d y_{i}, \quad \sigma_{\Phi}(x, y, z)=\sqrt{\sum_{j} \frac{\lambda_{j}}{V_{j}} \int_{-0.5 L}^{0.5 L} \varphi_{j}^{2}\left(y_{i}, x, y, z\right) d y_{i}}
$$

where $\varphi_{j}\left(y_{i}, x, y, z\right)$ is the concentration of pollutants from the moving sources of the $j$ th lane ( $j$ th type) with an exhaust power $q_{j}$, velocity $V_{j}$, and intensity $\lambda_{j}$.

The distribution of the mathematical expectation (7) of the waste concentration in the vertical plane $y=0$ located at an identical distance from the ends of the road sector considered depends on the road length; as follows from (4) and (5), this dependence decreases with increasing road length. To find the representative length $L$ of the road sector sufficient for evaluating the distribution of the waste concentration in a given region, for example, $G_{1}=\{(x, z) \mid x \in[10,200], z \in[0,100]\}(x \geqslant 0$ to avoid a singularity at the point $x=0, z=b$ ), we construct the functional $I(L)$, which determines the maximum difference in two solutions (mathematical expectations) within the entire domain considered for two values $L$ and $L+\Delta L$ :

$$
\begin{equation*}
\left.I(L)=\left.\frac{\lambda}{V} \max _{x, z \in G_{1}}\right|_{-0.5 L-\Delta L} ^{0.5 L+\Delta L} \varphi\left(y_{i}, x, 0, z\right) d y_{i}-\int_{-0.5 L}^{0.5 L} \varphi\left(y_{i}, x, 0, z\right) d y_{i} \right\rvert\, \tag{10}
\end{equation*}
$$

The value of $L$ for which this functional attains the minimum value yields an objective estimate of the least length of the road sector; with further increase in this length, the change in the mathematical expectation of the waste concentration is negligibly small. The road sector of this length may be considered to be representative, and the data obtained on this sector may be used for estimating the concentration of pollutants on the road as a whole.

We study the possibility of approximation of unsteady transport and dispersion of gaseous wastes produced by randomly appearing cars in a spatial domain by the process of diffusion of pollutants from a straight-line stationary source of constant intensity, which is described by the two-dimensional differential equation


Fig. 2. Dependence of the waste concentration on time for a random one-way single-lane flux of cars at different distances $x$ from the road.

$$
U \cos \alpha \frac{\partial \varphi}{\partial x}+W \frac{\partial \varphi}{\partial z}=K_{x} \frac{\partial^{2} \varphi}{\partial x^{2}}+K_{z} \frac{\partial^{2} \varphi}{\partial z^{2}}+Q \delta(x, z-b)
$$

with the boundary conditions $\varphi \rightarrow 0, x \rightarrow \pm \infty, z \rightarrow \infty$, and $\partial \varphi(x, 0) / \partial z=0$, where $Q$ is the sought power of the straight-line source of pollutants, which approximates the overall exhaustion of motor transport. The solution of this equation has the form [12]

$$
\begin{align*}
\varphi(x, z)= & \frac{Q}{2 \pi \sqrt{K_{x} K_{z}}}\left[K_{0}\left(0.5 \sqrt{\frac{U^{2} \cos ^{2} \alpha}{K_{x}}+\frac{W^{2}}{K_{z}}} \sqrt{\frac{x^{2}}{K_{x}}+\frac{(z-b)^{2}}{K_{z}}}\right) \exp \left(\frac{x U \cos \alpha}{2 K_{x}}+\frac{(z-b) W}{2 K_{z}}\right)\right. \\
& \left.+K_{0}\left(0.5 \sqrt{\frac{U^{2} \cos ^{2} \alpha}{K_{x}}+\frac{W^{2}}{K_{z}}} \sqrt{\frac{x^{2}}{K_{x}}+\frac{(z+b)^{2}}{K_{z}}}\right) \exp \left(\frac{x U \cos \alpha}{2 K_{x}}-\frac{(z+b) W}{2 K_{z}}\right)\right], \tag{11}
\end{align*}
$$

where $K_{0}$ is the MacDonald function. We represent Eqs. (7) and (11) in the form

$$
M_{\Phi}(x, 0, z)=\frac{q \lambda}{4 \pi V \sqrt{K_{x} K_{y} K_{z}}} \omega(x, z), \quad \varphi(x, z)=\frac{Q}{2 \pi \sqrt{K_{x} K_{z}}} \theta(x, z),
$$

where the form of the functions $\omega(x, z)$ and $\theta(x, z)$ is determined by comparing these expressions with (5)-(7) and (11), respectively. To estimate the measure of closeness of the functions $M_{\Phi}$ and $\varphi$, we use the squared norm

$$
J=\left\|M_{\Phi}-\varphi\right\|_{G_{1}}^{2}=\frac{Q^{2}}{4 \pi^{2} K_{x} K_{z}}\|\theta\|_{G_{1}}^{2}-\frac{q Q \lambda}{4 \pi^{2} V K_{x} K_{z} \sqrt{K_{y}}}(\theta, \omega)+\frac{q^{2} \lambda^{2}}{16 \pi^{2} V^{2} K_{x} K_{y} K_{z}}\|\omega\|_{G_{1}}^{2} .
$$

The minimum of $J$ corresponds to the value of $Q$ for which the distribution of pollutants from a random flux of cars in the domain $G_{1}$ is approximated most accurately. Denoting

$$
(\theta, \omega)=\int_{G_{1}} \theta(x, z) \omega(x, z) d x d z, \quad\|\theta\|_{G_{1}}^{2}=\int_{G_{1}} \theta^{2}(x, z) d x d z,
$$

we obtain

$$
\begin{equation*}
\frac{d J}{d Q}=0, \quad Q=\frac{q \lambda(\theta, \omega)}{2 V \sqrt{K_{y}}\|\theta\|_{G_{1}}^{2}} \tag{12}
\end{equation*}
$$

In numerical investigation, it is assumed $[1,14,15]$ that the length of the road sector considered is $L=1000 \mathrm{~m}$, the distance between the sources and the road is $b=0.5 \mathrm{~m}$, the traffic intensity is $\lambda=0.5 \mathrm{sec}^{-1}$, the angle is $\alpha=0$, the velocity of cars is $V=12.5 \mathrm{~m} / \mathrm{sec}$, the air-flow velocity is $U=3 \mathrm{~m} / \mathrm{sec}$, the deposition rate of carbon oxide is $W=0$, the coefficient of turbulent diffusion is $K_{x}=K_{y}=67 \mathrm{~m}^{2} / \mathrm{sec}$ and $K_{z}=26 \mathrm{~m}^{2} / \mathrm{sec}$, and the power of point sources is $q_{\mathrm{CO}}=0.12 \cdot 10^{-3} \mathrm{~kg} / \mathrm{sec}$.


Fig. 3. Convergence of the values of the mean concentration (1) and its root-mean-square deviation (2) to the exact values of $M_{\Phi}(3)$ and $\sigma_{\Phi}(4)$.


Fig. 4. Distributions of the mathematical expectation $M_{\Phi}$ (a) and root-mean-square deviation of the waste concentration $\sigma_{\Phi}(\mathrm{b})$ for $L=1000 \mathrm{~m}$ in a part of the domain $G_{1}$.

The appearance of cars at the initial point of the road is simulated by a generator of the steady Poisson process. Based on the known velocity $V$, the positions of all point sources of pollutants of power $q$ on the road are traced for each time, and the corresponding fields of waste concentrations are determined, after which the fields of concentrations are summed. As a consequence, the concentrations of pollutants at all points of the domain are described by random functions. Some realizations of these functions (for points located in the vertical plane $y=0$ at a height of 2 m from the surface and distances of 10,25 , and 50 m from the road) are shown in Fig. 2.

For one of the points $(x=10 \mathrm{~m}, y=0, z=2 \mathrm{~m})$, we determine the mean concentrations $\bar{\Phi}=$ $\frac{1}{t} \int_{0}^{t} \Phi(\tau) d \tau$ and root-mean-square deviations of the waste concentration $s_{\Phi}=\left[\frac{1}{t} \int_{0}^{t}(\Phi(\tau)-\bar{\Phi})^{2} d \tau\right]^{1 / 2}$ from the mean value $\bar{\Phi}$ using realizations of the random process of one-way, single-lane motion (Fig. 3). These results demonstrate the convergence $\bar{\Phi} \rightarrow M_{\Phi}$ and $s_{\Phi} \rightarrow \sigma_{\Phi}$ as $t \rightarrow \infty$ and allow us to estimate the time of establishing of the mean value of the concentration close to the true value of the mathematical expectation. For the conditions considered, this time is about 30 min (the deviation of $\bar{\Phi}$ from the exact value $M_{\Phi}$ is within $2.5 \%$ ).

The calculations of functional (10) for $\Delta L=10 \mathrm{~m}$ showed that $I(L)=0.889 \cdot 10^{-3} \mathrm{mg} / \mathrm{m}^{3}$ and $I(L+$ $\Delta L)=0.881 \cdot 10^{-3} \mathrm{mg} / \mathrm{m}^{3}$ already for $L=1000 \mathrm{~m}$ (the difference is less than $1 \%$ ). Hence, it is sufficient

TABLE 1

| $\alpha, \operatorname{deg}$ | $f$ | $\alpha, \operatorname{deg}$ | $f$ |
| :---: | :---: | :---: | :---: |
| 45 | 0.02 | 225 | 0.05 |
| 90 | 0.10 | 270 | 0.24 |
| 135 | 0.06 | 315 | 0.10 |
| 180 | 0.12 | 360 | 0.16 |



Fig. 5. Distribution of the mathematical expectation of the concentration of heavy pollutants from a random flux of cars: curves 1 and 2 refer to results obtained for a constant wind direction and taking into account the wind rose, respectively.
to consider a sector of this length to simulate a steady emission of pollutants to the region under study. The distributions of the mathematical expectation $M_{\Phi}$ and root-mean-square deviation $\sigma_{\Phi}$ of the waste concentration in a part of the domain $G_{1}$ for $L=1000 \mathrm{~m}$ are plotted in Fig. 4.

To estimate the possibility of approximation of the mathematical expectation $M_{\Phi}(x, 0, z)$ determined by Eq. (7) by the distribution $\varphi(x, z)$ of the concentration of pollutants from a straight-line stationary source of power $Q$, we use formula (12). The calculations for $G_{1}=\{(x, z) \mid x \in[10,200], z \in[0,100]\}$ yield the value $Q=7.1824 \cdot 10^{-6} \mathrm{~kg} /(\mathrm{m} \cdot \mathrm{sec})$. It should be noted that the function $M_{\Phi}(x, 0, z)$ cannot be identically replaced by the distribution of the concentration of pollutants $\varphi(x, z)$ from a straight-line source of constant power.

Finally, we consider the transport, dispersion, and deposition of lead compounds exhausted by a fourlane flux of cars. The mean rate of exhaustion of heavy lead-containing particles is estimated in [16] by the value $W=-0.002 \mathrm{~m} / \mathrm{sec}$. According to [15], we have $q_{\mathrm{Pb}}=1.2 \cdot 10^{-3} \mathrm{~kg} / \mathrm{sec}$ and $\lambda=2 \mathrm{sec}^{-1}$. The rest of the parameters remain unchanged. The distribution of wind directions during a year (the wind rose for the road sector considered [17]) is given in Table 1 ( $f$ is the relative frequency). The distribution of the mathematical expectation of the concentration of lead compounds on the soil surface $(z=0)$ for a constant wind direction $(\alpha=0)$ taking into account this wind rose is plotted in Fig. 5.

Thus, in the present paper we formulated a spatial problem of transport and dispersion of light and heavy pollutants produced by moving cars whose random flux is assumed to be described by the Poisson process. An approximate solution of this problem is also given. Using a numerical experiment, it was found that the mathematical expectation of the concentration of pollutants may be considered to be timeindependent for road sectors of large length and significant time intervals. This allows us to approximate the unsteady process of waste emission from randomly appearing cars by a steady model with a source of pollutants of constant power distributed continuously along the road. The model presented allows us to justify the minimum necessary time of field measurements during which the value of the concentration approaches the value of the mathematical expectation.

## REFERENCES

1. O. V. Rodivilova, V. V. Kostrov, L. V. Shvedova, and E. V. Krivtsova, "Contamination of the atmosphere of Ivanovo by the waste gases of automobile transport," Inzh. Ékolog., No. 4, 100-107 (1996).
2. V. I. Tarankov and S. M. Matveev, Effect of Automobile Contamination on Pine Plantations in Voronezh [in Russian], Voronezh Forest Technology Inst., Voronezh (1992). Deposited at VNIITslesresursy 10.26.92, No. 910-lkh92.
3. O. D. Volkova and T. S. Samoilova, "Methodology of ecological normalization of automobile waste loads on forest ecosystems," in: Ecological Normalization: Problems and Methods [in Russian], Moscow (1992), pp. 35-37.
4. D. P. Chock, "A simple line-source model for dispersion near roadways," Atmos. Environ., 12, No. 4, 823-829 (1978).
5. A. K. Luhar and R. S. Patil, "A general finite line source model for vehicular pollution prediction," Atmos. Environ., 23, No. 3, 555-562 (1989).
6. G. T. Csanady, "Crosswind shear quality model performance - a summary of the AMS workshop on dispersion model performance," Bull. Amer. Meteorol. Soc., 61, 599-609 (1981).
7. R. Sivacoumar and K. Thanasekaran, "Line source model for vehicular pollution prediction near roadways and model evaluation through statistical analysis," Environ. Pollut., 104, No. 2, 389-395 (1998).
8. A. K. Luhar and R. S. Patil, "Estimation of emission factors for Indian vehicles," Indian J. Air Pollut. Control, 7, 155-160 (1986).
9. K. C. Heidorn, A. E. Davies, and M. C. Murphy, "Wind tunnel modelling of roadways: comparison with mathematical models," J. Air Waste Managment Assoc., 41, No. 11, 1469-1475 (1991).
10. Y. Moriguchi and K. Uehara, "Numerical and experimental simulation of vehicle exhaust gas dispersion for complex urban roadways and their surroundings," J. Wind. Eng., 25, No. 2, 102-107 (1987).
11. P. S. Kasibhatla, L. K. Peters, and G. Fairweather, "Numerical simulation of transport from an infinite line source: Error analysis," Atmos. Environ., 22, No. 1, 75-82 (1988).
12. G. I. Marchuk, Mathematical Simulation in Environmental Problems [in Russian], Nauka, Moscow (1982).
13. E. S. Ventsel' and L. A. Ovcharov, Theory of Random Processes and Its Engineering Applications [in Russian], Nauka, Moscow (1991).
14. I. G. Filippov, V. G. Gorskii, and T. N. Shvetsova-Shilovskaya, "Dispersion of an admixture in the near-Earth layer of the atmosphere," Teor. Osn. Khim. Tekhnol., 29, No. 5, 517-521 (1995).
15. A. V. Ruzskii, V. V. Donchenko, V. A. Petrukhin, et al., Technique for Calculating Automobile Wastes Exhausted into the Atmosphere on City Roads [in Russian], Research Inst. "Atmosphere," Moscow (1996).
16. G. M. Chernogaeva, V. A. Petrukhin, and S. A. Gromov, "Balance of pollutants in river basins of some background regions of the USSR," in: Monitoring of Background Contamination of Natural Media [in Russian], Gidrometeoizdat, Leningrad (1990), No. 6, pp. 171-174.
17. Conditions of Environment and Health of the Perm' Population in 1997: Reference Book [in Russian], Perm' Committee on Environmental Protection, Perm' (1998).
